



Binfield C of E School Calculation Policy

Last Edited 27th January 2019



Where should I start?

This calculation policy is based on a developmental approach to the four operations. All children should begin with stage 1 and build up to stage 4. Children should only move on to the next stage once they have 'mastered' their current stage. To have 'mastered' a stage the children should be able to:

- calculate confidently using the methods at that stage
- confidently explain the purpose of each calculation
- explain each stage in the calculation
- use concrete resources, pictorial methods **and** abstract methods to complete the calculation.

It may help to ask your child or their teacher what methods they are currently using in their class.

As a rough guide, we would expect that most children's abilities would follow the table below. It is however only a guide: mastering the calculations at each stage is essential and is more critical to progressing their learning than moving as quickly as possible through the stages. Because understanding in mathematics is cumulative, it can be counter-productive to move children to the next stage if they have not mastered the previous stage.

Stage	Developing mastery of this stage is expected in:
Stage 1	YR/Y1
Stage 2	Y1/Y2
Stage 3	Y2/Y3
Stage 4	Y3/Y4
Stage 5	Y4/Y5

Mastery of all stages achieved	Y5/Y6
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		National Curriculum Statement	
		Number and place value	Addition and Subtraction
		Multiplication and Subtraction	
Ye ar 1	<ul style="list-style-type: none"> count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number count, read and write numbers to 100 in numerals; count in multiples of 2s, 5s and 10s given a number, identify 1 more and 1 less identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least read and write numbers from 1 to 20 in numerals and words 	<ul style="list-style-type: none"> read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs represent and use number bonds and related subtraction facts within 20 add and subtract one-digit and two-digit numbers to 20, including 0 solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = ? - 9$ 	<ul style="list-style-type: none"> solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
Ye ar 2	<ul style="list-style-type: none"> count in steps of 2, 3, and 5 from 0, and in 10s from any number, forward and backward recognise the place value of each digit in a two-digit number (10s, 1s) identify, represent and estimate numbers using different representations, including the number line compare and order numbers from 0 up to 100; use <, > and = signs read and write numbers to at least 100 in numerals and in words use place value and number facts to solve problems 	<ul style="list-style-type: none"> solve problems with addition and subtraction: <ul style="list-style-type: none"> using concrete objects and pictorial representations, including those involving numbers, quantities and measures applying their increasing knowledge of mental and written methods recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 add and subtract numbers using concrete objects, pictorial representations, and mentally, including: <ul style="list-style-type: none"> a two-digit number and 1s a two-digit number and 10s 2 two-digit numbers adding 3 one-digit numbers show that addition of 2 numbers can be done in any order (commutative) and subtraction of 1 number from another cannot recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems 	<ul style="list-style-type: none"> recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs show that multiplication of 2 numbers can be done in any order (commutative) and division of 1 number by another cannot solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

		National Curriculum Statement	
		Number and place value	Addition and Subtraction
			Multiplication and Subtraction
Year 3	<ul style="list-style-type: none"> count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number recognise the place value of each digit in a 3-digit number (100s, 10s, 1s) compare and order numbers up to 1,000 identify, represent and estimate numbers using different representations read and write numbers up to 1,000 in numerals and in words solve number problems and practical problems involving these ideas 	<ul style="list-style-type: none"> add and subtract numbers mentally, including: <ul style="list-style-type: none"> a three-digit number and 1s a three-digit number and 10s a three-digit number and 100s add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction estimate the answer to a calculation and use inverse operations to check answers solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction 	<ul style="list-style-type: none"> recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects
Year 4	<ul style="list-style-type: none"> count in multiples of 6, 7, 9, 25 and 1,000 find 1,000 more or less than a given number count backwards through 0 to include negative numbers recognise the place value of each digit in a four-digit number (1,000s, 100s, 10s, and 1s) order and compare numbers beyond 1,000 identify, represent and estimate numbers using different representations round any number to the nearest 10, 100 or 1,000 solve number and practical problems that involve all of the above and with increasingly large positive numbers read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of 0 and place value 	<ul style="list-style-type: none"> add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate estimate and use inverse operations to check answers to a calculation solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why 	<ul style="list-style-type: none"> recall multiplication and division facts for multiplication tables up to 12×12 use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers recognise and use factor pairs and commutativity in mental calculations multiply two-digit and three-digit numbers by a one-digit number using formal written layout solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

		National Curriculum Statement	
		Number and place value	Addition and Subtraction
			Multiplication and Subtraction
Ye ar 5	<ul style="list-style-type: none"> • read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit • count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000 • interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through 0 • round any number up to 1,000,000 to the nearest 10, 100, 1,000, 10,000 and 100,000 • solve number problems and practical problems that involve all of the above • read Roman numerals to 1,000 (M) and recognise years written in Roman numerals 	<ul style="list-style-type: none"> • add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) • add and subtract numbers mentally with increasingly large numbers • use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why 	<ul style="list-style-type: none"> • identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers • know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers • establish whether a number up to 100 is prime and recall prime numbers up to 19 • multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers • multiply and divide numbers mentally, drawing upon known facts • divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context • multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 • recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3) • solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes • solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign • solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates

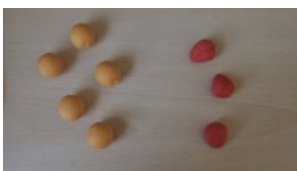
		National Curriculum Statement	
		Addition and Subtraction	Multiplication and Subtraction
Ye ar 6	<ul style="list-style-type: none"> • read, write, order and compare numbers up to 10,000,000 and determine the value of each digit • round any whole number to a required degree of accuracy • use negative numbers in context, and calculate intervals across 0 • solve number and practical problems that involve all of the above 	<ul style="list-style-type: none"> • multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context • divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context • perform mental calculations, including with mixed operations and large numbers • identify common factors, common multiples and prime numbers • use their knowledge of the order of operations to carry out calculations involving the 4 operations • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why • solve problems involving addition, subtraction, multiplication and division • use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy 	

ADDITION

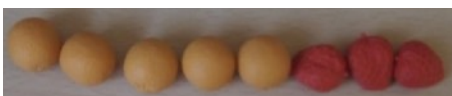
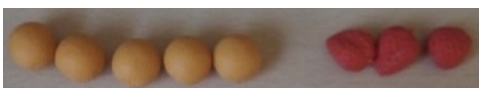
	Concrete and visual support	Abstract support
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Children use counting objects to show how parts add together to make a whole.

$$5 + 3 = 8$$
$$3 + 5 = 8$$



Stage 1



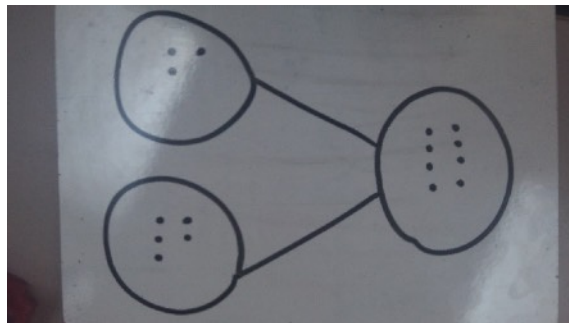
Or:



Children use counting objects to show how parts add together to make a whole – regrouping the objects into groups of tens.

Children use part-part-whole diagrams to demonstrate addition of two numbers.

$$5 + 3 = 8$$
$$3 + 5 = 8$$



Children use counters to assist in simple additions, without regrouping.

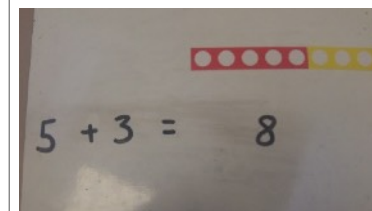
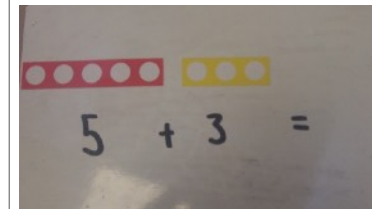
[Video: Addition counters – no regrouping](#)

Children use part-part-whole diagrams to demonstrate additions, showing parts and wholes grouped into tens and ones.

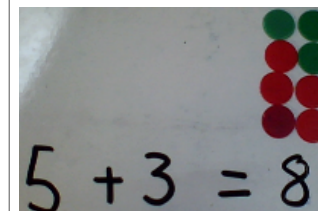
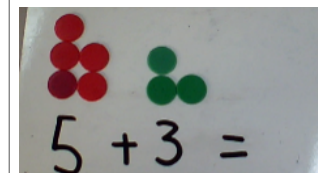
$$7 + 8 = 15$$

Children use counters to add two 1 digit numbers, without regrouping.

With Carrbarrs



With counters:

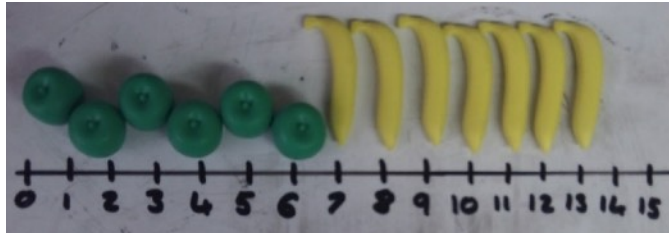


Children use counters to add two 1 digit numbers, regrouping.

Stage 1

Children count on using number lines and counting objects, unifix, numicon etc

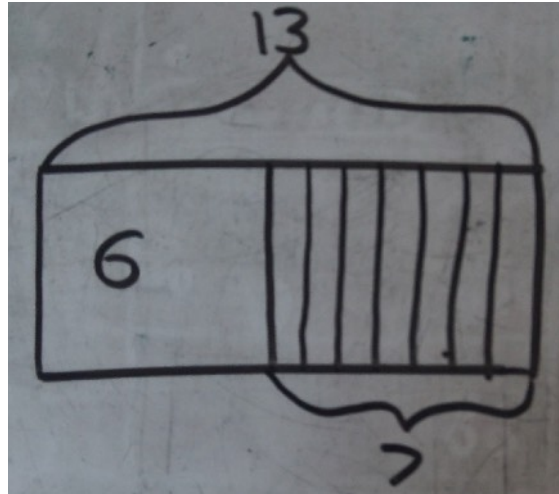
$$6 + 7 = 13$$



Stage
2

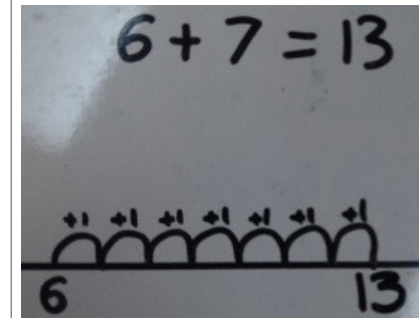
Children draw a bar model in which they can count on rather than count all.

$$6 + 7 = 13$$



Children use a number line to add two 1 digit numbers.

Counting from left to right.

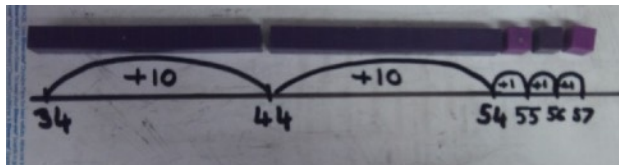


Stage 3

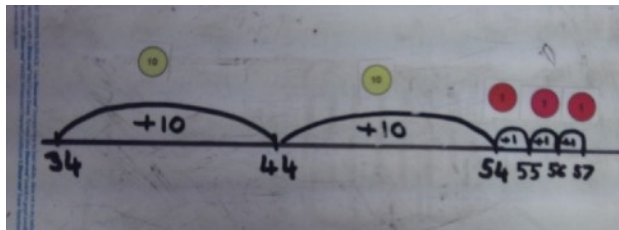
Children count on using number lines, with Diennes and place value counters, in which children can see each ten being added separately followed by each one.

$$34 + 23 = 57$$

Using Diennes:

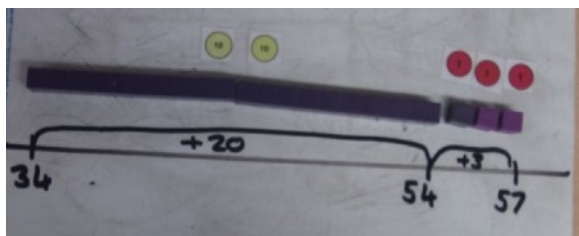


Using place value counters:



Children count on using number lines, with Diennes and using place value counters, in which tens are added as a group, followed by ones as a group.

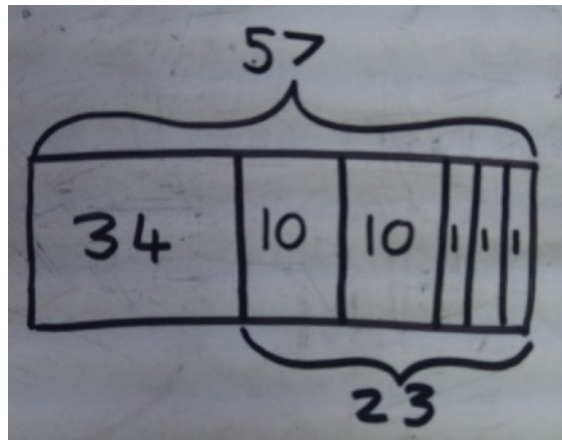
$$34 + 23 = 57$$



Children use Diennes or place value counters to show how groups of ten are added, and then a group of ones is taken away to compensate.

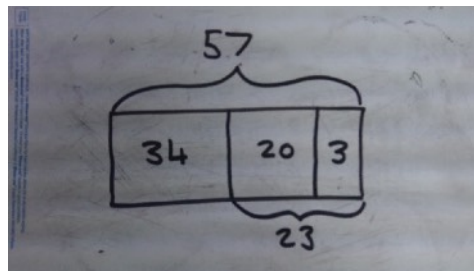
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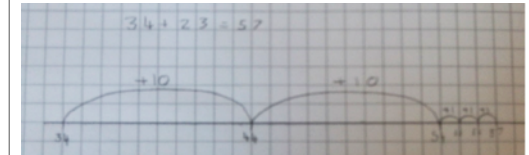
Children use a bar model in which children can see the numbers being added in groups of tens and groups of one.

$$34 + 23 = 57$$

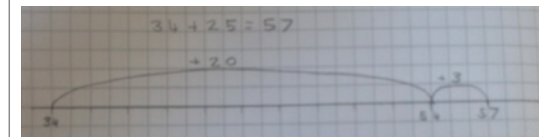


Children use a bar model in which children can see

Children use a number line to add two 2 digit numbers, first counting in tens, then in ones.



Children use a number line to add two 2 digit numbers, adding the tens in one jump, then the ones in one jump.



[Video: Numberline addition – groups of 10](#)

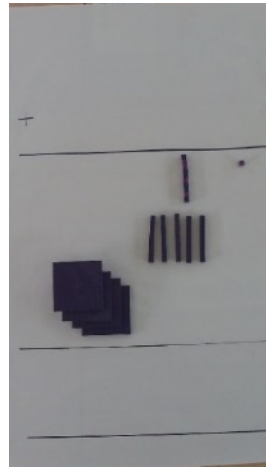
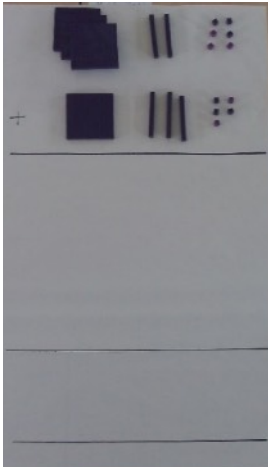
[Video: Numberline addition – tens then ones](#)

Stage 3

Children use Diennes or place value counters to represent the expanded column method of addition.

$$326 + 235 = 561$$

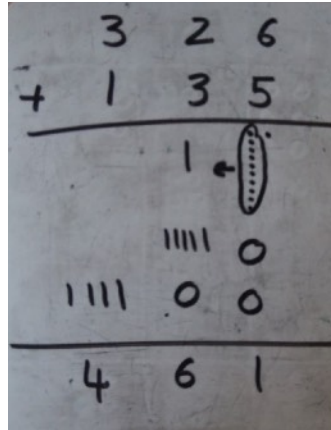
Using Diennes:



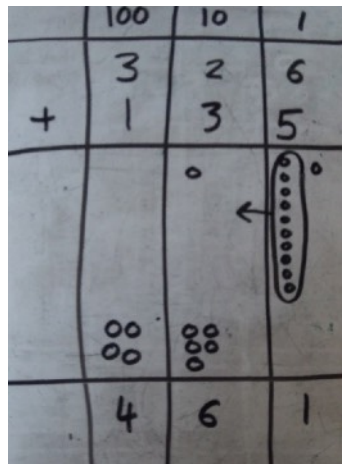
Children draw Diennes or place value counters to represent the expanded column method of addition.

$$326 + 137 = 461$$

Drawing Diennes:

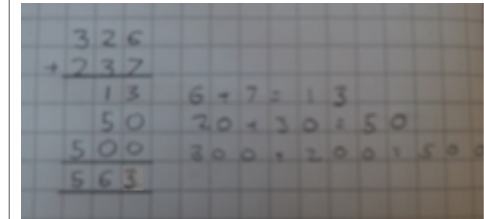


Drawing place value counters:



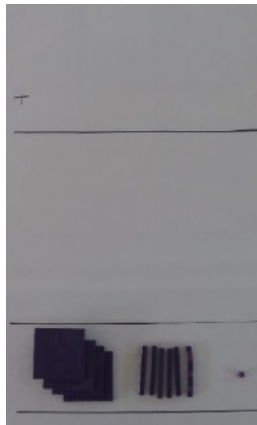
Children use the expanded column method of addition to add 2 and 3 digit numbers, beginning with the least significant digit first.

$$326 + 437 = 563$$



[Video: Expanded column addition](#)

Stage 4

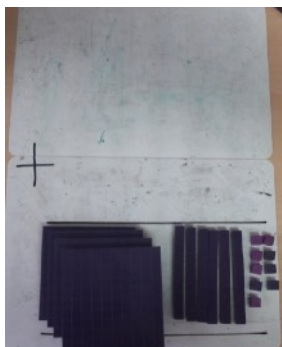
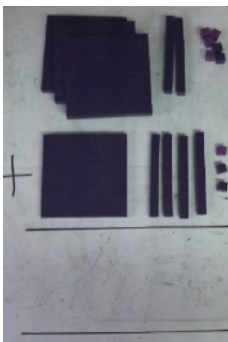


Using place value counters:

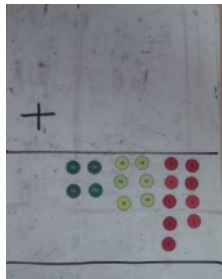
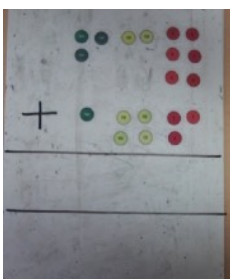
Children use Diennes or place value counters to represent the column method of addition.

$$326 + 143 = 469$$

Using Diennes

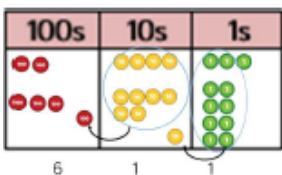


Using place value counters:



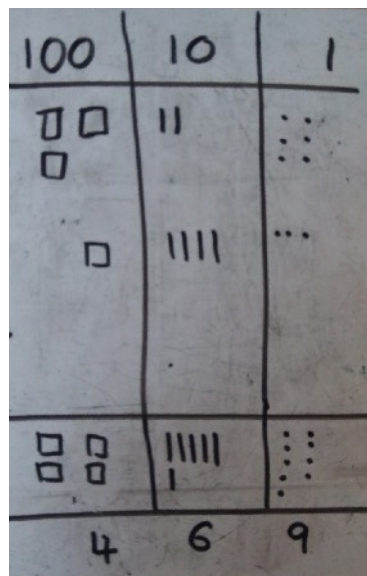
Children use place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- children exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.

$$243 + 368 = 611$$



Children draw Diennes or place value counters to represent the column method of addition, without carrying below the line.

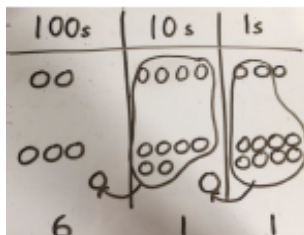
$$326 + 143 = 469$$



[Video: Column addition – no regrouping](#)

Children represent the counters in a place value chart, circling when they make an exchange.

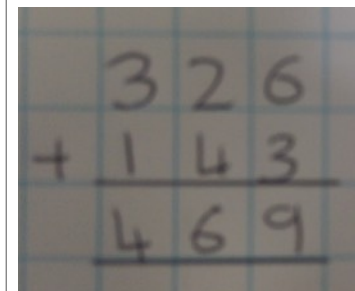
$$243 + 368 = 611$$



[Video: Column addition - regrouping](#)

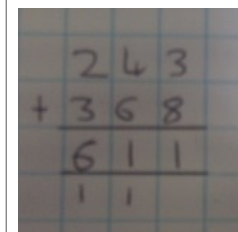
Children use the column method of addition to add 2, 3 and 4 digit numbers without needing to carry below the line.

$$326 + 143 = 469$$

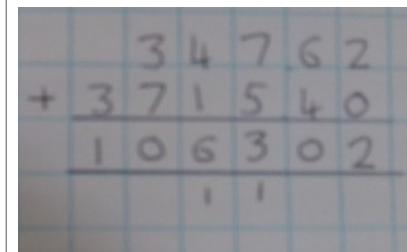


Children use the column method of addition to add 2, 3 and 4 digit numbers, carrying below the line.

$$243 + 368 = 611$$

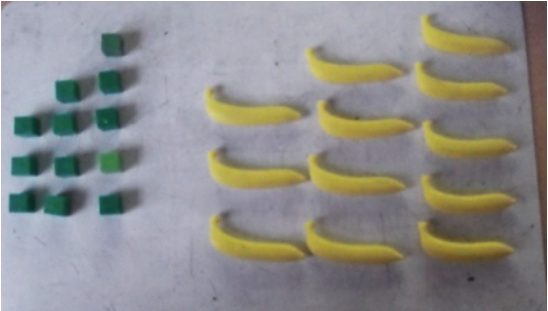
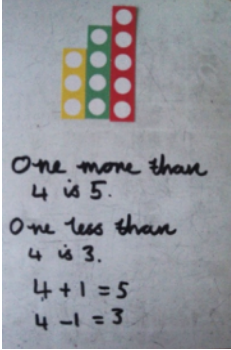


$$347.62 + 3715.40 = 106302$$



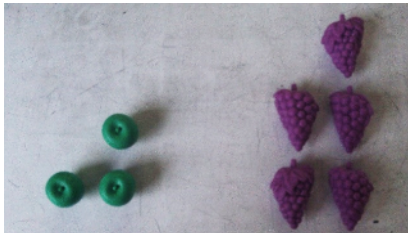
[Video: Adding decimals](#)

SUBTRACTION

	Concrete and visual support	Abstract Calculation
	<p data-bbox="495 284 1400 316">Children use counting objects to demonstrate one more and one less.</p> <p data-bbox="692 347 1202 379">What is one more and one less than 3?</p>  <p>The image shows two groups of objects on a light-colored surface. On the left, there are 4 green cubes arranged in a 2x2 grid. On the right, there are 9 yellow bananas arranged in three rows of three.</p>	<p data-bbox="1704 284 2063 347">Children find one more and one less</p>  <p>The image shows a vertical stack of ten blocks: 4 yellow, 2 green, and 4 red. Below the blocks, the following text is written in cursive:</p> <p data-bbox="1800 523 1995 571">One more than 4 is 5.</p> <p data-bbox="1800 584 1966 632">One less than 4 is 3.</p> <p data-bbox="1823 644 1928 676">$4 + 1 = 5$</p> <p data-bbox="1823 679 1921 711">$4 - 1 = 3$</p>

Children use counting objects to show how a whole can be made into parts.

The whole: 8, can be split into parts: 3 and 5.



Children use counting objects to show how a whole can be split into parts - regrouping the objects into groups of tens.

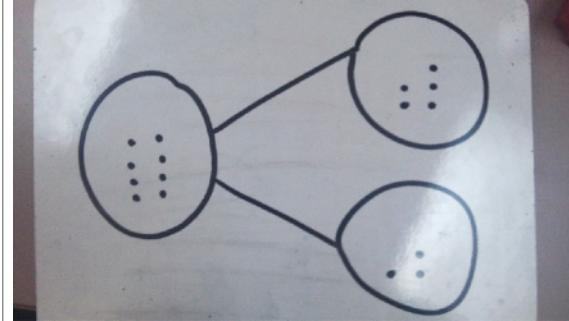
$$15 - 7 = 8$$

$$15 - 8 = 7$$



Children use part-part-whole diagrams to demonstrate subtraction of two numbers.

The whole: 8, can be split into parts: 3 and 5.



Children use counters to assist in simple subtractions, without regrouping.

[Video: Column subtraction – no regrouping](#)

Children use part-part-whole diagrams to demonstrate subtractions, showing parts and wholes grouped into tens and ones.

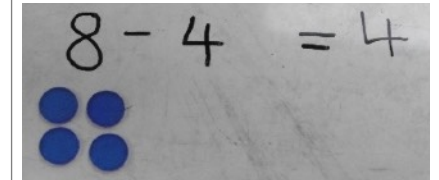
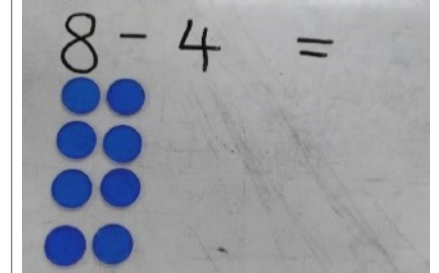
$$15 - 7 = 8$$

$$15 - 8 = 7$$

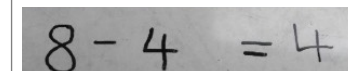
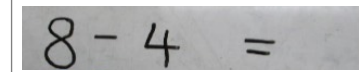


Children use counters to subtract two 1 digit numbers without regrouping.

With counters:



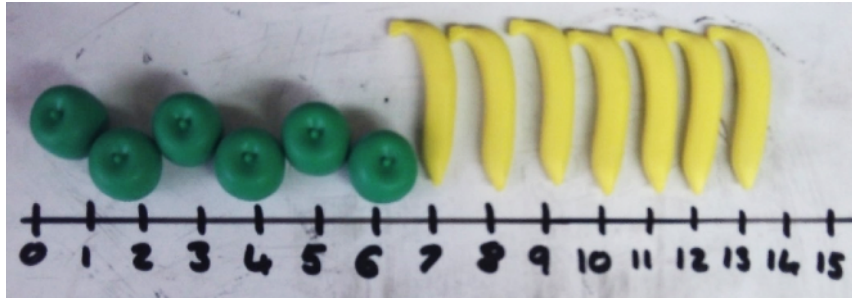
With Carrbars



Children use counters to subtract two 1 digit numbers, regrouping.

Children count back using number lines and counting objects, unifix, numicon etc

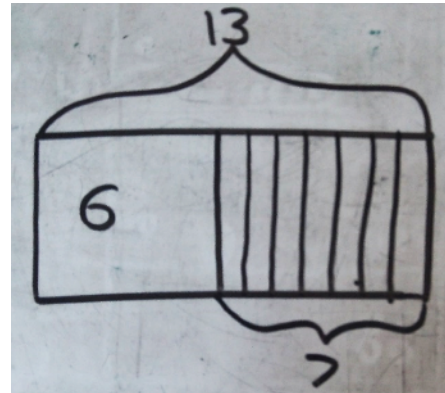
$$13 - 6 = 7$$



Stage 2

Children draw a bar model in which they can count back in ones. .

$$13 - 6 = 7$$



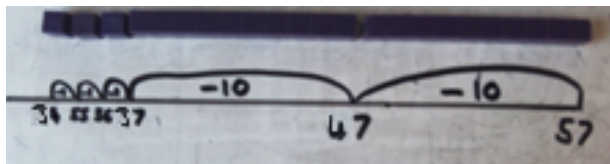
Children use a number line to subtract 1 digit numbers.

[Video: Numberline subtraction](#)

Children count back using number lines, with Diennes and place value counters, in which children can see the tens being subtracted separately followed by the ones.

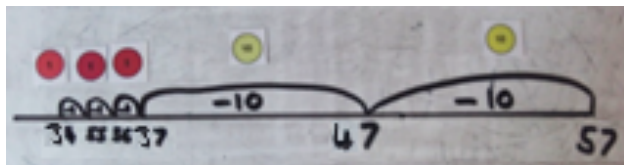
$$57 - 23 = 34$$

With Diennes:



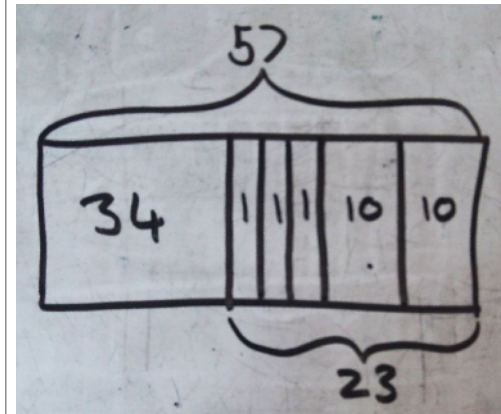
Stage 3

With place value counters:



Children use a bar model in which children can see each tens being subtracted separately, followed by each one.

$$57 - 23 = 34$$

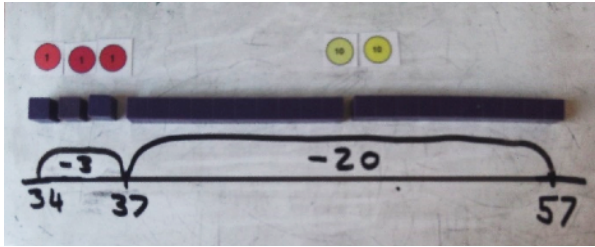


Children use a number line to subtract 2 digit numbers, first with tens, then in ones.

[Video: Numberline subtraction - groups of 10](#)

Children count on using number lines, with Diennes and using place value counters, in which the tens are subtracted as a group, followed by the ones as a group.

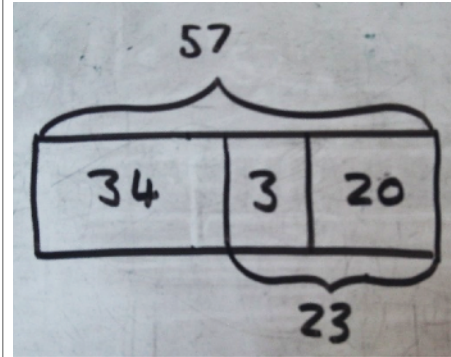
$$57 - 23 = 34$$



[Video: Numberline subtraction – tens then ones](#)

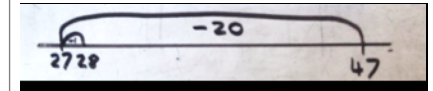
Children use a bar model in which they can see the numbers being subtracted in groups of tens and groups of one.

$$57 - 23 = 34$$



Children use a number line to subtract 2 digit numbers, subtracting the tens in one jumps, then the ones in one jump.

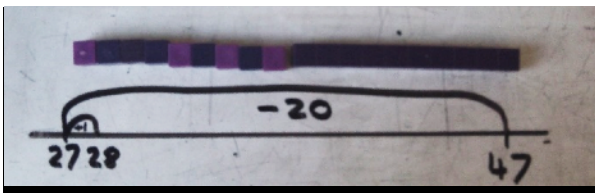
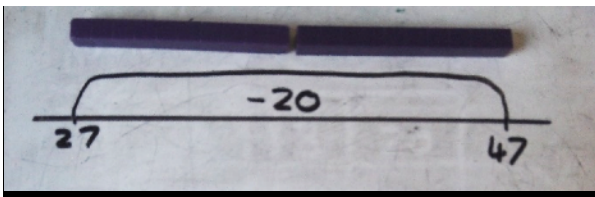
$$57 - 23 = 23$$



Stage 3

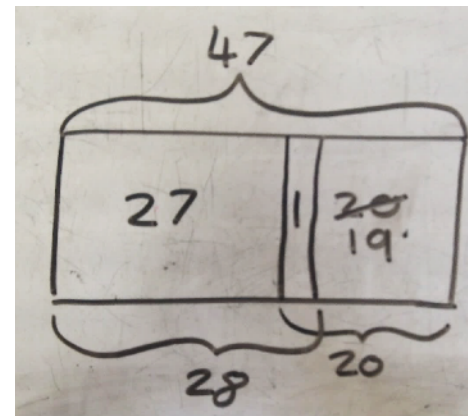
Children use Diennes or place value counters to show how groups of ten are subtracted, and then a group of ones is added to compensate.

$$47 - 19 = 38$$



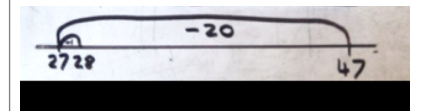
Children use a bar model in which children can see the tens being subtracted and then ones added to compensate.

$$47 - 19 = 28$$



Children use a number line to subtract 2 digit numbers, using the compensation method.

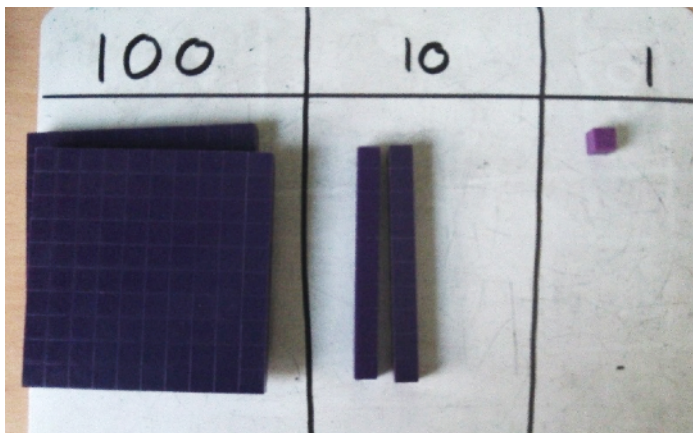
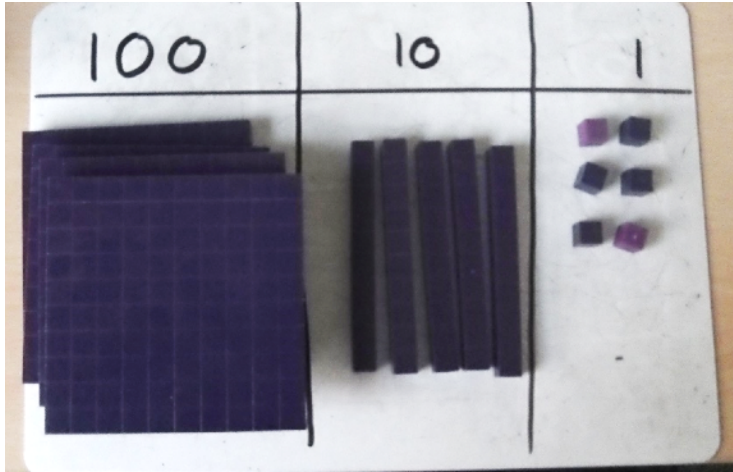
$$47 - 19 = 28$$



[Video: Numberline subtracton compensation](#)

Children use Diennes or place value counters to represent the column method of subtraction, without needing to exchange.

$$456 - 235 = 221$$

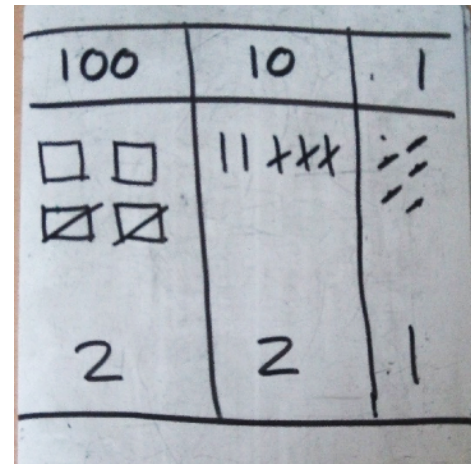


Stage
4

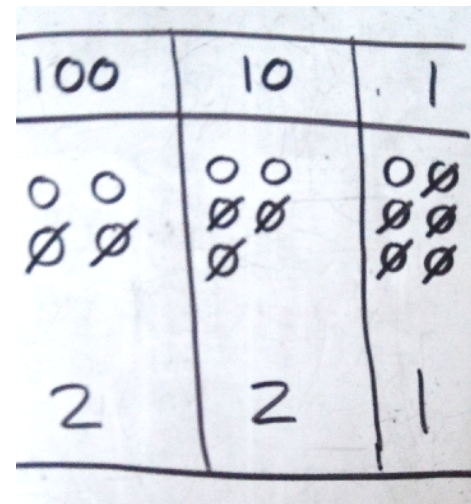
Children draw Diennes or place value counters to represent the column method of subtraction, without needing to exchange.

$$456 - 235 = 221$$

Drawing Diennes:

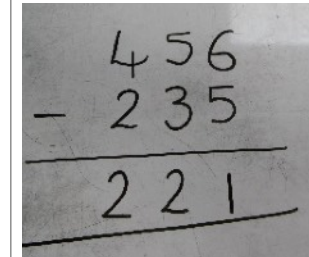


Drawing place value counters:



Children use the column method of subtraction to subtract 2 digit and 3 digit numbers, without needing to exchange.

$$456 - 235 = 221$$



Hint: We say 'exchange' because we exchanging one lot of ten for ten lots of one. We don't 'borrow' because in this calculation we will never give anything back!

[Video: Column subtraction – no carrying](#)

MULTIPLICATION

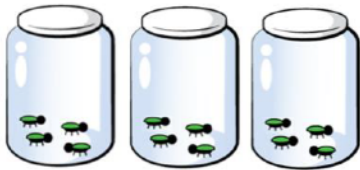
	Concrete and Pictorial Support	Abstract Method
--	---------------------------------------	------------------------

Children show repeated addition using objects, or unifix.

There are 3 equal groups, with 4 in each group.

Children understand this is the same as:

$$4 + 4 + 4 = 12$$



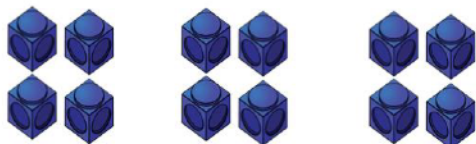
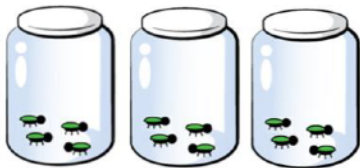
Stage 1

Children show repeated addition using counting objects, or unifix.

There are 3 equal groups, with 4 in each group.

Children understand this is the same as:

$$4 \times 3 = 12$$

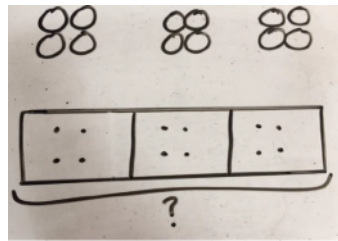


Children to represent the practical resources in a picture and use a bar model.

There are 3 equal groups, with 4 in each group.

Children understand this is the same as:

$$4 + 4 + 4 = 12$$

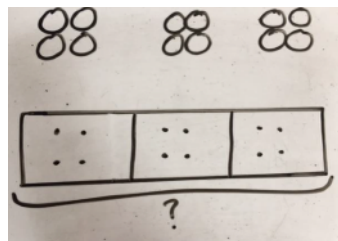


Children to represent the practical resources in a picture and use a bar model.

There are 3 equal groups, with 4 in each group.

Children understand this is the same as:

$$4 \times 3 = 12$$

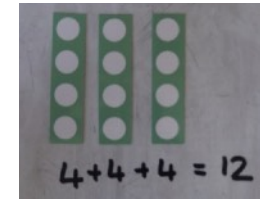


Children can use repeated addition for 2, 5, 10 using 'groups of' phrasing and Carrbars to show multiplication.

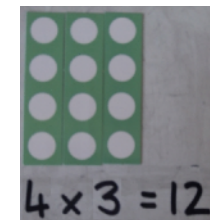
There are 3 equal groups, with 4 in each group.

Children understand this is the same as:

$$4 + 4 + 4 = 12$$



Children can use a written method involving the vocabulary '3 groups of four'.



Hint: We say that this the calculation represented here is:

'three lots of four is 12'.

The notation we use to show this is:

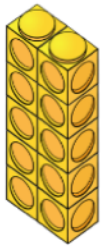
$$4 \times 3 = 12.$$

This is because we take four objects and multiply them three times. 4 is the multiplicand, 3 is the multiplier and 12 is the product.

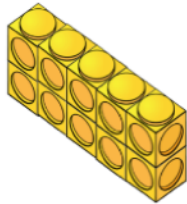
Stage
1

Children create arrays to illustrate commutativity.
Counters and other objects can also be used.

$$5 \times 2 = 10$$
$$2 \times 5 = 10$$



2 lots of 5



5 lots of 2

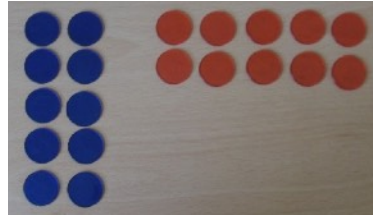
Hint: The word 'commutative' comes from 'commute' or 'move around', so the commutative property refers to being able to move numbers around within number sentences.

For example: when 5 is the multiplicand and 2 is the multiplier, we get the product 10: written $5 \times 2 = 10$.
When the multiplicand is 2 and the multiplier is 5 the product is still 10: written $2 \times 5 = 10$.

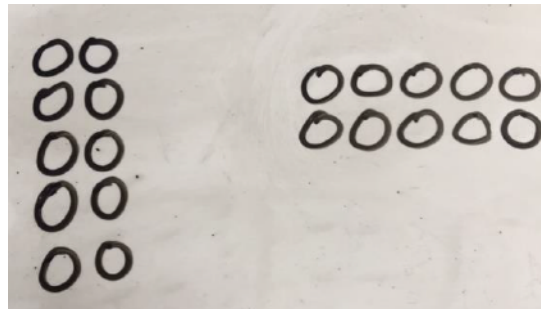
Children create arrays using counters, and pictorially.

$$5 \times 2 = 10$$
$$2 \times 5 = 10$$

With counters:

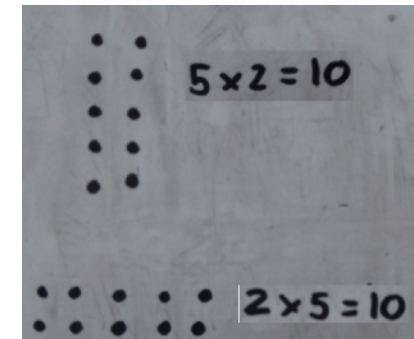


Drawing counters:



Children can use arrays to show repeated addition.

$$5 \times 2 = 10$$
$$2 \times 5 = 10$$

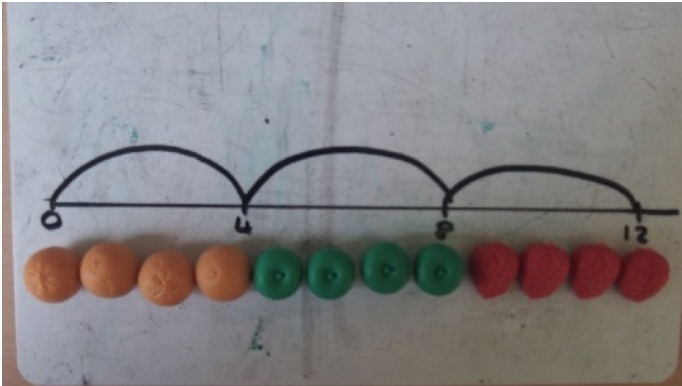


[Video – Multiplication arrays](#)

Stage
2

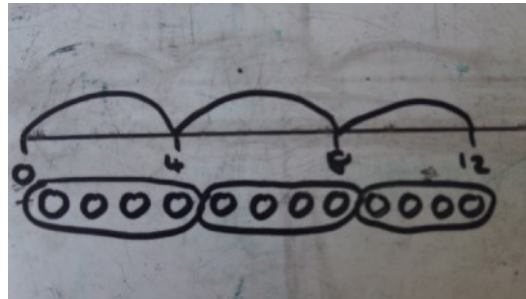
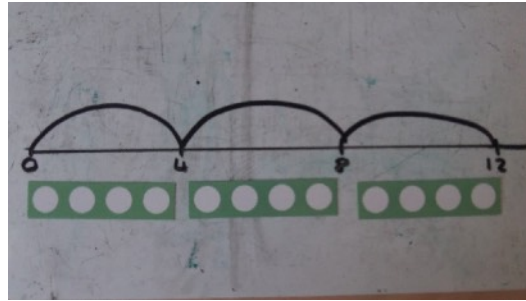
Children to represent repeated addition on a number line using counting objects or Carrbars.

$$4 \times 3 = 12$$



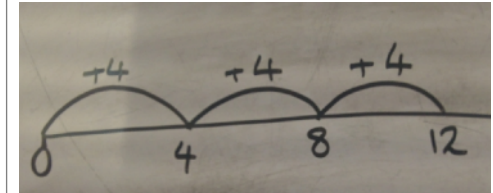
Children to represent repeated addition pictorially on a number line.

$$4 \times 3 = 12$$



Children can use number lines to show multiplication as repeated addition.

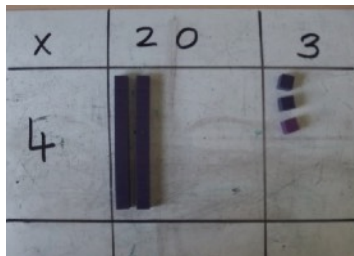
$$4 \times 3 = 12$$



[Video: Multiplication – repeated addition – number line](#)

Children calculate using the grid method using Diennes or place value counters.

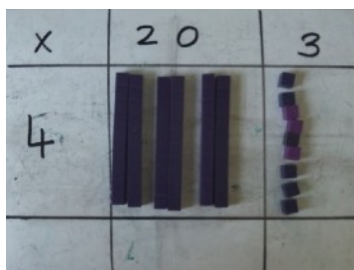
$$23 \times 4 = 92$$



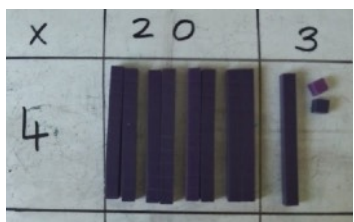
Step 1. =23



Step 2. =46



Step 3. =69

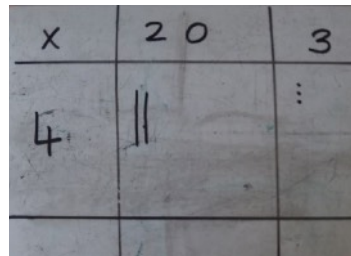


Step 4. =92

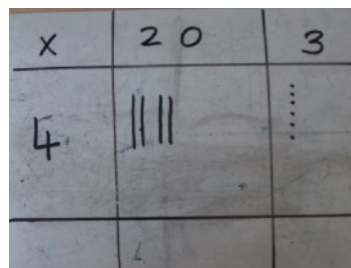
Stage 3

Children represent the grid method by drawing Diennes or place value counters.

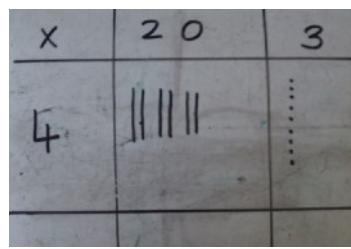
$$23 \times 4 = 92$$



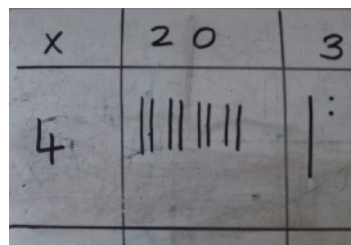
Step 1. =23



Step 2. =46



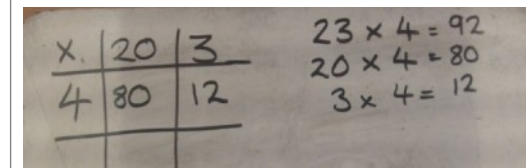
Step 3. =69



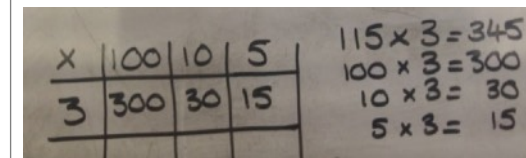
Step 4. =92

Children can use partitioning through the grid method to multiply.

$$23 \times 4 = 92$$



$$3 \times 115 = 345$$

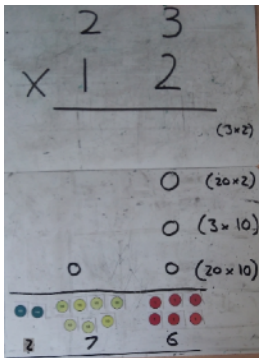
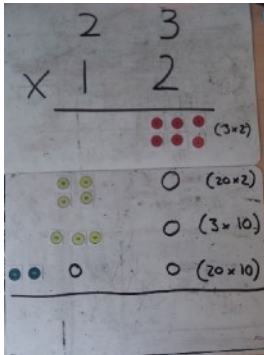


[Video: Multiplication – grid method](#)

Stage 4

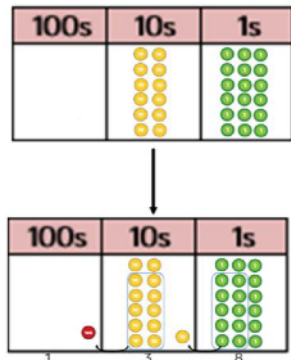
Children calculate using the expanded column method using place value counters.

$$23 \times 12 = 267$$



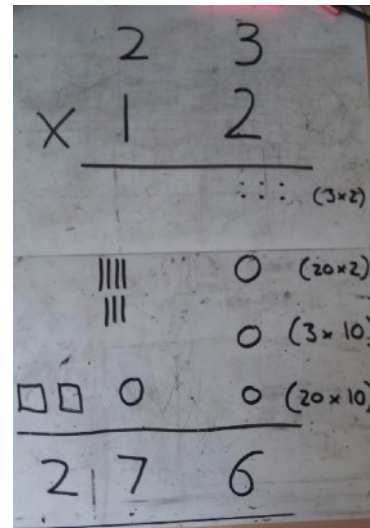
Children use compact column method with place value counters.

$$23 \times 6 = 138$$



Children represent the expanded column method by drawing Diennes.

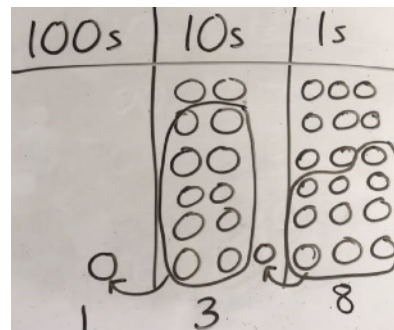
$$23 \times 12 = 267$$



Hint: We complete this calculation using the same method as left.

Children represent the counters/base 10, pictorially e.g. the image below.

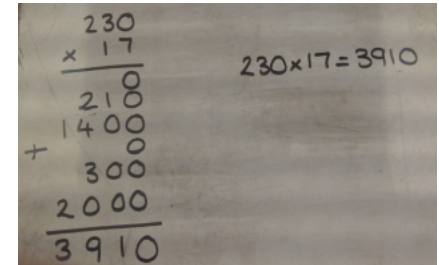
$$23 \times 6 = 138$$



[Video: Multiplication – expanded column method](#)

Children can use expanded column method to multiply 2 and 3 digit numbers.

$$230 \times 17 = 3910$$



Hint: The expanded column method leads children to the more compact method so that they understand its structure and efficiency.

Children can use the compact method to multiply 2, 3 and 4 digit numbers.

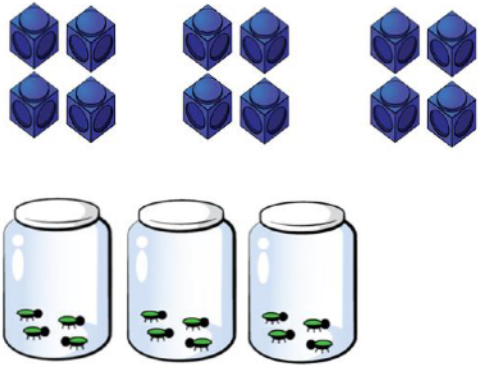
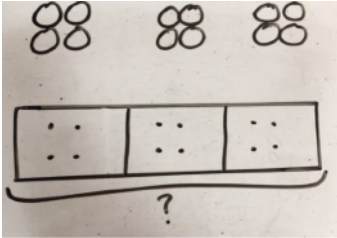
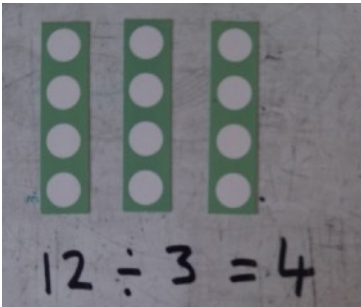
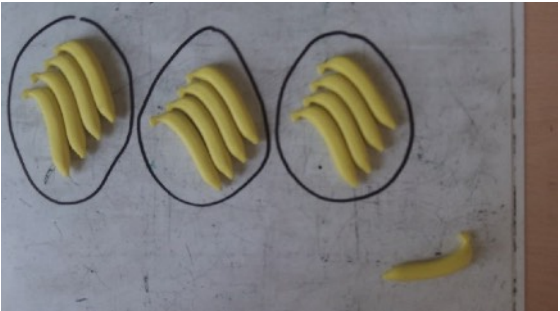
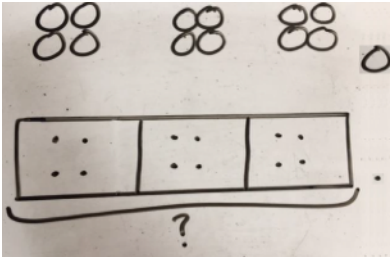
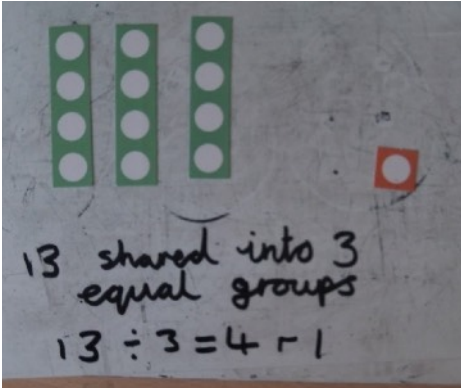
$$23 \times 6 = 138$$

$$6 \times 23 =$$

$$\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ 11 \end{array}$$

Hint: The compact column method is similar to the expanded column method, in that we break the calculation down in the same way. In this example we begin by finding 6 lots of three ($3 \times 6 = 18$). We write the 8 lots of one in the ones column and carry the 1 lot of ten in the tens column below the line. We then find 6 lots of 20 ($20 \times 6 = 120$). We write the 2 lots of ten, together with the 1 lot of ten we had previously carried below the line, and

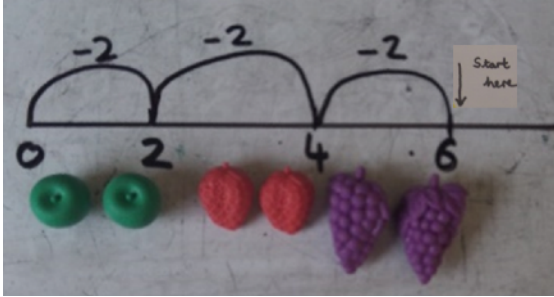
DIVISION

	Concrete and abstract support		Pictorial
Stage 1	<p>Children share objects into groups, recording representations, without remainders.</p> <p>$12 \div 3 = 4$</p>  <p>The concrete representation shows 12 blue cubes arranged in three groups of four. Below them are three jars, each containing four green ants, representing the division of 12 objects into 3 equal groups.</p>	<p>Children show objects being shared into groups using bar models, without remainders.</p> <p>$12 \div 3 = 4$</p>  <p>The abstract representation shows 12 circles arranged in three groups of four. Below them is a bar model divided into three equal sections, each containing four dots, with a question mark below the bar.</p>	<p>Children share objects into groups, recording representations, without remainders.</p>  <p>The pictorial representation shows three vertical green strips, each with four white dots. Below the strips is the equation $12 \div 3 = 4$ written in black ink.</p>
Stage 1	<p>Children share objects into groups, recording representations, with remainders.</p> <p>$13 \div 3 = 4 \text{ r}1$</p>  <p>The concrete representation shows 13 bananas arranged in three groups of four, with one banana left over. The groups are circled in black, and the remaining banana is separate.</p>	<p>Children show objects being shared into groups using bar models, with remainders.</p> <p>$13 \div 3 = 4 \text{ r}1$</p>  <p>The abstract representation shows 13 circles arranged in three groups of four, with one circle left over. Below them is a bar model divided into three equal sections, each containing four dots, with a question mark below the bar.</p>	<p>Children share objects into groups, recording representations, with remainders.</p>  <p>The pictorial representation shows three vertical green strips, each with four white dots, and one red square. Below the strips is the equation $13 \text{ shared into } 3 \text{ equal groups}$ and $13 \div 3 = 4 \text{ r}1$ written in black ink.</p>

Stage
2

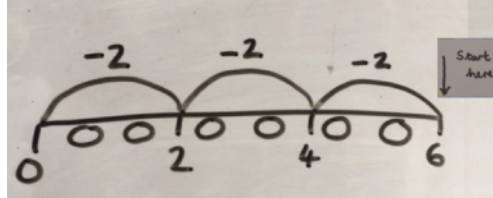
Children to represent repeated subtraction on a number line using concrete resources, without remainders.

$$6 \div 3 = 3$$

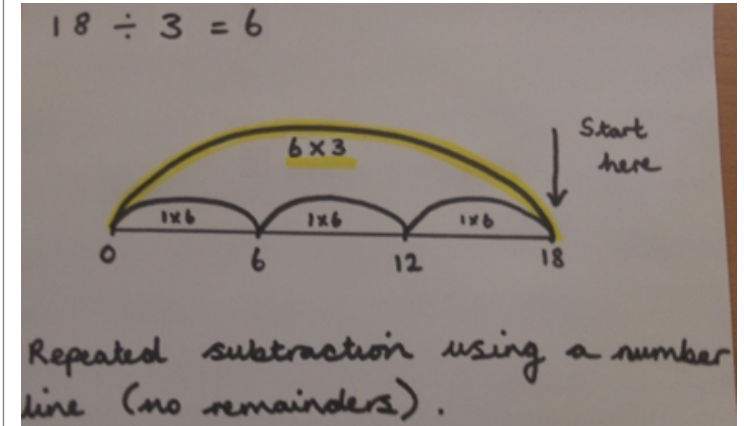


Children to represent repeated subtraction pictorially on a number line, without remainders.

$$6 \div 3 = 2$$

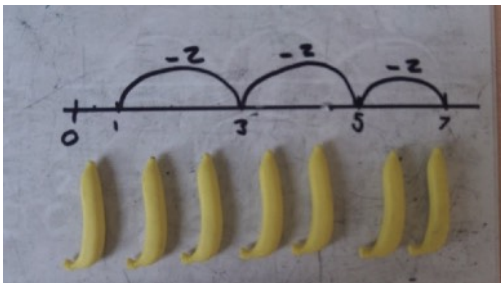


Children show division as repeated subtraction, using a number line, without remainders.



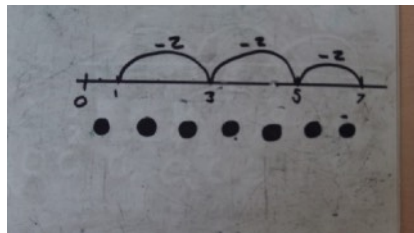
Children to use repeated subtraction on a number line using counting objects, with remainders.

$$7 \div 3 = 2 \text{ r}1$$



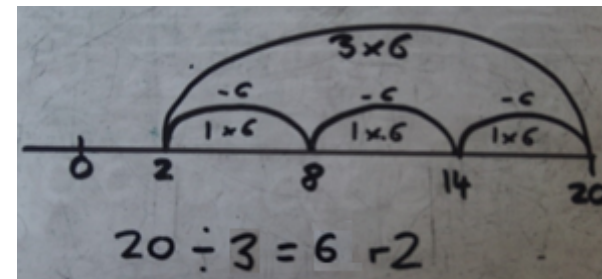
Children to represent repeated subtraction pictorially on a number line, with remainders.

$$7 \div 3 = 2 \text{ r}1$$



Children show division as repeated subtraction, using a number line, with remainders.

$$20 \div 3 = 6 \text{ r}1$$



Stage 2

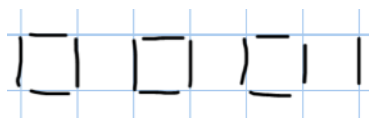
Children perform divisions by making shapes using lolly sticks. Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.



There are 3 whole squares, with 1 left over.

$$13 \div 3 = 4 \text{ r}1$$

Children to represent the lollipop sticks pictorially.



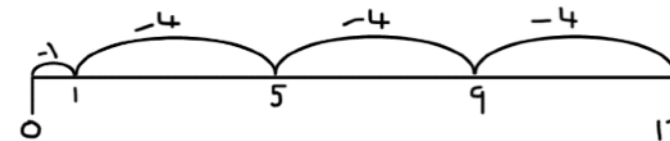
There are 3 whole squares, with 1 left over.

$$13 \div 3 = 4 \text{ r}1$$

Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'

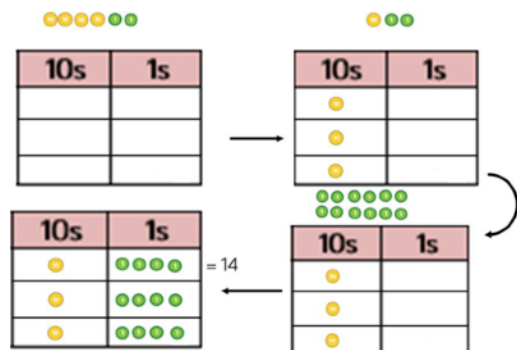
$$13 \div 3 = 4 \text{ r}1$$



Children represent the chunking method with Diennes or place value counters, on a number line, without remainders.

Sharing using place value counters.

$$42 \div 3 = 14$$

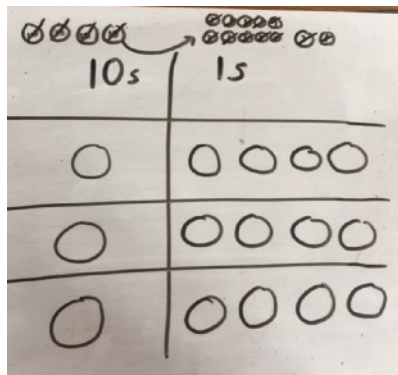


Stage 3

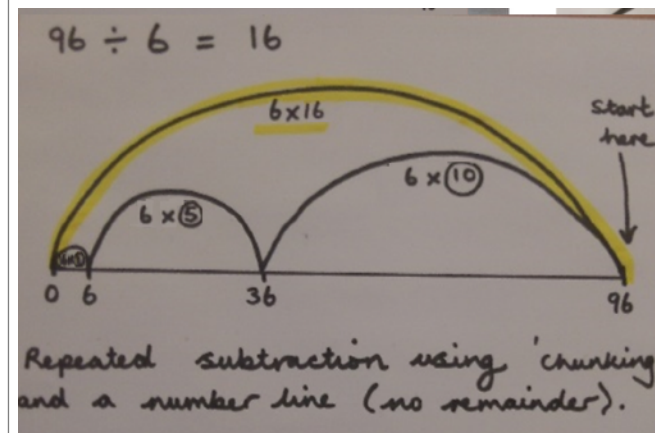
Children represent the chunking method pictorially, on a number line, without remainders.

Children to represent the place value counters pictorially.

$$42 \div 3 = 14$$



Children use the chunking method on a number line without remainders.



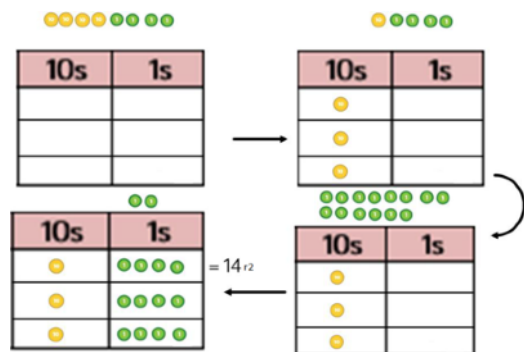
Hint: Chunking is a method used for dividing larger numbers that are difficult to divide mentally. Chunking is repeated subtraction of the divisor and multiples of the divisor – in other words, working out how many groups of a number fit into another number. In this example we:

- begin with 96 and subtract 10 lots of 6 to get 36
- we then subtract 5 lots of 6 to get 30
- next we subtract a last lot of 6 to get 0
- finally we add together the multipliers – 10, 5 and 1 to get 16
- thus we know that $96 \div 6 = 16$

Children represent the chunking method with Diennes or place value counters, on a number line, with remainders.

Sharing using place value counters.

$$44 \div 3 = 14 \text{ r}2$$

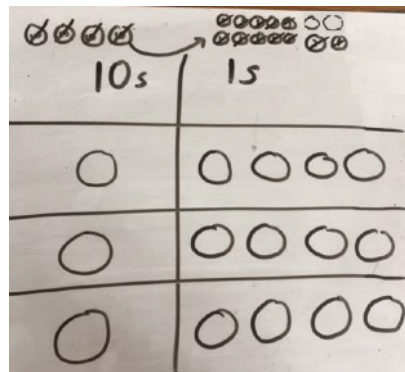


Stage 3

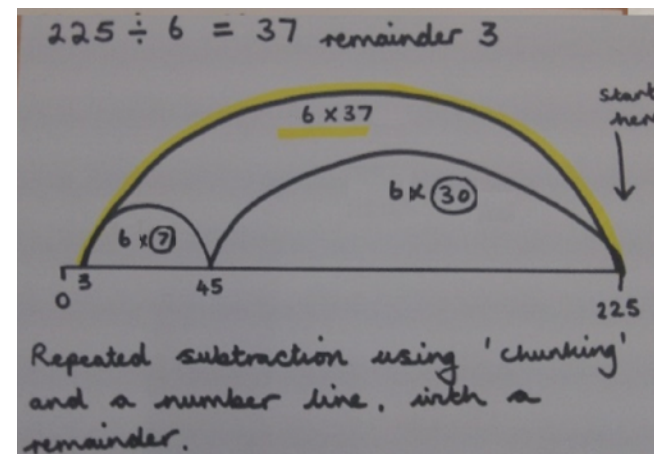
Children represent the chunking method pictorially, on a number line, with remainders.

Children to represent the place value counters pictorially.

$$44 \div 3 = 14 \text{ r}2$$



Children use the chunking method on a number line with remainders.



Hint: Chunking can be used to perform division calculations with or without remainders. In this example we:

- begin with 225 and subtract 30 lots of 6 to get 45
- we then subtract 7 lots of 6 to get 3, from which we cannot subtract a group of 6
- finally we add together the multipliers – 30 and 7 to get 37.
- thus we know that $225 \div 6 = 37 \text{ r}3$

Sta
ge
4

Children use the column chunking method without remainders.

The image shows a handwritten column chunking method for the division $96 \div 6 = 16$. The calculation is written on a grey background. At the top, the equation $96 \div 6 = 16$ is written. Below it, the column subtraction is shown: 96 minus 60 (labeled (6×10)) equals 36 . Then, 36 minus 36 (labeled (6×6)) equals 0 . At the bottom, the final result is written as $10 + 6 = 16$.

Hint: Chunking can be used to perform division calculations without use of a number line. Here for example we:

- begin with 96 and subtract 10 lots of 6 to get 36
- we then subtract 6 lots of 6 to get 0
- finally we add together the multipliers – 10 and 6 to get 36.
- thus we know that $96 \div 6 = 16$

Sta
ge
4

Children use the column chunking method with remainders.

Handwritten work showing the column chunking method for $279 \div 13 = 21 \text{ r}6$. The work is as follows:

$$\begin{array}{r} 279 \\ - 130 \quad (13 \times 10) \\ \hline 149 \\ - 130 \quad (13 \times 10) \\ \hline 19 \\ - 13 \quad (13 \times 1) \\ \hline 6 \end{array}$$

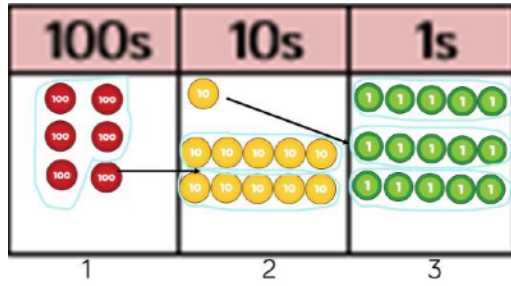
Below the subtraction, the multipliers are summed: $10 + 10 + 1 = 21$.

Hint: Chunking can be used to perform division calculations without use of a number line whether or not there is a remainder. Here for example we:

- begin with 279 and subtract 10 lots of 13 to get 149
- we then subtract another 10 lots of 13 to get 19
- next we subtract another lot of 13 to get 6, from which we cannot subtract another lot of 13
- finally we add together the multipliers – 10 and 6 to get 36.
- finally we add together the multipliers – 10, 10 and 1 to get 21
- thus we know that $279 \div 13 = 21 \text{ r}6$

Children complete short division using place value counters to group.

$$615 \div 5 = 123$$

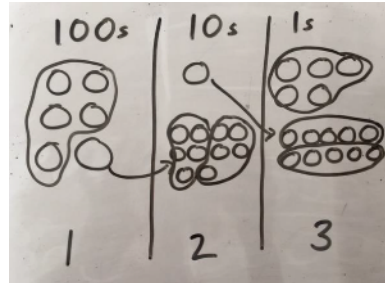


Stage
5

1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Children represent the place value counters pictorially.

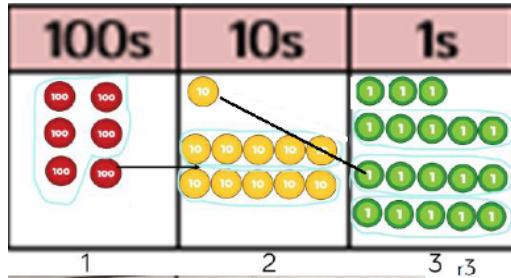
$$615 \div 5 = 123$$



Children use short division, without remainders.

Children complete short division using place value counters to group.

$$618 \div 5 = 123 \text{ r}3$$

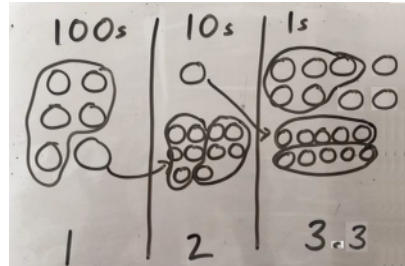


Stage
5

1. Make 618 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 18 ones?
7. What is the remainder?

Children represent the place value counters pictorially.

$$618 \div 5 = 123 \text{ r}3$$



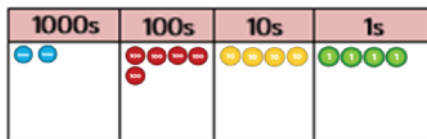
Children use short division converting remainders to fractions and decimals.

$$376 \div 5 = 75 \frac{1}{5}$$

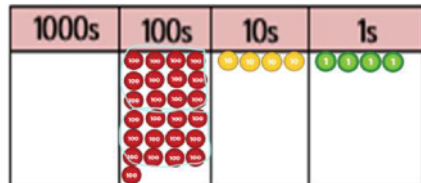
Children complete long division using place value counters to group.

$$2544 \div 12 = 212$$

Stage 5

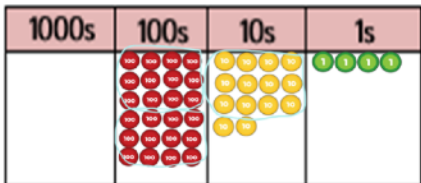


We can't group 2 thousands into groups of 12 so will exchange them.



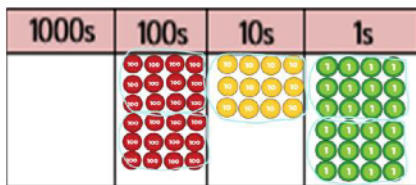
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

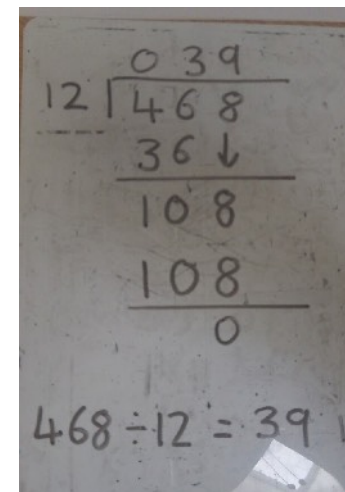


After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 groups of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Children use long division, without remainders.

$$468 \div 12 = 39$$



[Video: Long division](#)

